Кафедра електроніки
Факультет аеронавігації, електроніки та телекомунікацій (ФАЕТ)

Електронні системи

Electronic Systems

Lecture #3

Яновський, Фелікс Йосипович
професор, доктор технічних наук,
лауреат Державної премії України, IEEE Fellow
Завідувач кафедри електроніки НАУ

Орієнтовний тематичний план лекцій

Основи теорії систем, сигнали і первинні перетворювачі електронних систем

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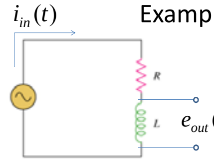
System Analysis – Аналіз систем

- Now we consider the math tool that is necessary for analysis of any linear system and investigation of its operation.
- All input and output values of a system depend on time. That is why in this tool time-variable (t) should be used.
- We are interested in determination an output value, if the input value is known. Then, we should seek for a ***transfer function*** - *передавальна функція*.
- Does it depend on the type of the input value (driving signal) or just on the system itself?
- Let's consider an ***RL***-circuit and let we know the dependence

$$f(t) = \frac{e_{out}(t)}{i_{in}(t)}$$

Example: *RL*-circuit

$i_{in}(t)$



$e_{out}(t)$

If we know $f(t)$, let's clarify if we can find $e_{out}(t)$ at any $i_{in}(t)$ from the equation

$$e_{out}(t) = f(t) i_{in}(t)$$

Let's establish the relation between $e_{out}(t)$ and $i_{in}(t)$ using basic diff equations describing the components.

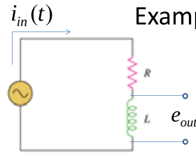
$e = Ri$ - resistor $\quad e = L\frac{di}{dt}$ - coil $\quad$ **Ohm law**

$$e_{out}(t) = L\frac{di_{in}}{dt}$$

Now we can find the solutions for $e_{out}(t)$ at given $i_{in}(t)$.

Example: *RL*-circuit

$i_{in}(t)$



$e_{out}(t)$

Let $i_{in}(t) = I_m \sin \omega t$ Then

$$e_{out}(t) = L\frac{di_{in}}{dt} = LI_m \omega \cos \omega t =$$
$$= \omega L I_m (\sin \omega t - \frac{\pi}{2})$$

Let's find ***transfer function*** in this special case:

$$f(t) = \frac{e_{out}(t)}{i_{in}(t)} = \frac{\omega L I_m \sin(\omega t - \pi/2)}{I_m \sin \omega t} = \frac{\omega L \sin(\omega t - \pi/2)}{\sin \omega t}$$

It is clear that this $f(t)$ **depends on the type of the input value**.
In case $i_{in}(t) = at$, the ratio $f(t)$ would be completely different.

Це не зручно!

Example: *RL*-circuit

It is clear that this $f(t)$ **depends on the type of the input value**.
In case $i_{in}(t) = at$, the ratio $f(t)$ would be completely different.

Це не зручно!

For effective system analysis we would like to find some general approach.

Для ефективного аналізу систем ми маємо знайти певний загальний підхід

For example, one can use a transform of the variable.

We would like to transform the expression for time $i_{in}(t) = I_m \sin \omega t$ using a new variable s .

Transformation of variables

Let's recollect the transformation of variables.

Equation: $(x+y)dx + dy = 0$ (1)

Change of variable: $y = v - x$ $dy = dv - dx$

Then instead of (1) we get: $\frac{dx + dv}{v-1} = 0$ (2)

The solution in variables x and v is $(v-1)e^x = C$

To return back to the solution for x and y we should make the inverse transform (here just a simple substitution): $y = Ce^{-x} - x + 1$

Another example of applying transformation methods is the transform of multiplication to addition ***log - antilog***

Introduction to Laplace transformation

- Transformation into a new variable s is similar by character to the considered two examples. However math tool is much more complicated.
- Importance of this transformation is demonstrated, considering a solution of the linear differential equation:

$$\frac{d^2 i(t)}{dt^2} + \frac{3di(t)}{dt} + 2i(t) = 0 \qquad (*)$$

- This eq can represent the sum of voltages in a closed el circuit. It can be solved by common classical method. But let's instead of that suppose that the general solution exists in the form: $i(t) = Ae^{st}$
- Variable or a parameter s may have any meaning, e.g. $s = j\omega$ then $i(t) = Ae^{j\omega t}$

Introduction to Laplace transformation

- The member $e^{j\omega t}$ is basically a sine signal: $e^{j\omega t} = \cos\omega t + j\sin\omega t$ imaginary part is just $\sin\omega t$
- If the supposed general solution is really acceptable, we just should find the meaning s for the given case.
- After substitution $i(t) = Ae^{st}$ into the eq (*) we get:

$$s^2 Ae^{st} + 3sAe^{st} + 2Ae^{st} = 0 \qquad (1)$$

- Pay attention that $(Ae^{st})' = Ase^{st}$ and $(Ae^{st})'' = As^2e^{st}$
- Eq (1) can be simplified:

$$(s^2 + 3s + 2)Ae^{st} = 0$$

and in supposition that $Ae^{st} \neq 0$ we have: $s^2 + 3s + 2 = 0$

Introduction to Laplace transformation

$s^2 + 3s + 2 = 0$ $s_1 = -2$ $s_2 = -1$

Then the concrete solution is $i(t) = A_1 e^{-2t} + A_2 e^{-t}$ where particular values s ($s_1 = -2$ and $s_2 = -1$) are included.

<u>Conclusion</u>:

1) We transformed the differential equation to a simpler algebraic equation;
2) Then we solved the algebraic equation;
3) Then we transformed the solution back to the expression for time $i(t)$; you can check that this eq is really the solution;

4) This order of actions can be applied, e.g. to eq. $e_{out}(t) = L\frac{di_{in}}{dt}$

Anyhow $e_{out}(t) \to E_{out}(s)$, $i_{in}(t) \to I_{in}(s)$

Laplace Transformation

- Transformation from time domain to s-domain is Laplace transformation:
  - $f(t)$ is function of time;
  - $F(s)$ is Laplace-image.

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)\, dt.$$

$$f(x) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{\sigma_1 - i\cdot\infty}^{\sigma_1 + i\cdot\infty} e^{sx} F(s)\, ds,$$

σ - деяке дійсне число (умови існування)

$$F(s) \leftrightarrow f(t)$$

Таблица преобразования Лапласа

№	Оригинал	Изображение	№	Оригинал	Изображение
1	$\delta(t)$	1	8	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
2	1	$\frac{1}{s}$	9	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
3	t	$\frac{1}{s^2}$	10	$e^{-\alpha t}\sin\omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
4	t^n $(n = 1, 2, \ldots)$	$\frac{n!}{s^{n+1}}$	11	$e^{-\alpha t}\cos\omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
5	$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	12	$\frac{1}{\alpha}(1-e^{-\alpha t})$	$\frac{1}{s(s+\alpha)}$
6	$t\,e^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$	13	$1(t-a)$	$\frac{1}{s}e^{-as}$
7	$t^n e^{-\alpha t}$	$\frac{1}{(s+\alpha)^{n+1}}$			

Application of Laplace transform in the theory of circuits

$$e_{out}(t) = L\frac{di_{in}}{dt} \qquad E_{out}(s) = sLI(s)$$

Member *di/dt* is just changed by $sI(s)$

$$E_{out}(s) \leftrightarrow e_{out}(t) \qquad I_{in}(s) \leftrightarrow i_{in}(t)$$

$$f(t) = \frac{e_{out}(t)}{i_{in}(t)}$$

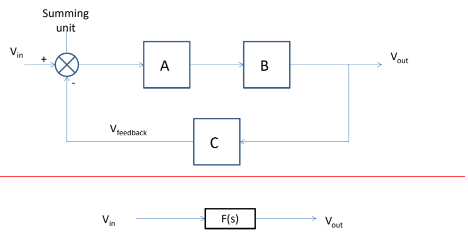
Transfer function — Передавальна функція

$$F(s) = \frac{E_{out}(s)}{I_{in}(s)}$$

Description of an electronic control system

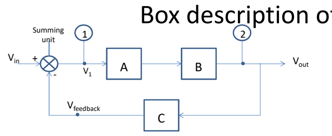
- A system can be described in differential equations or in transfer functions. Transfer function, being algebraic expression, is simpler in calculations and much more preferable for system analysis.
- Let's suppose that we deal with the system that consists of several sub-systems, that is, a number of black boxes.
- Each black box can be considered as a separate system that is described by its own transfer function.

Box description of systems – Блоковий опис систем



Система, що складається з трьох чорних ящиків та її еквівалент

Box description of systems



$$F_A(s) = 4$$

$$F_B(s) = \frac{s}{s^2+1}$$

$$F_C(s) = \frac{1}{s-2}$$

Now we can find a full transfer function that relates $V_{out}(s)$ with $V_{in}(s)$

1. From point 1 to 2:

$$G(s) = F_A(s)\cdot F_B(s) = 4\cdot\frac{s}{s^2+1} = \frac{4s}{s^2+1} = \frac{4s}{(s-j)(s+j)}$$

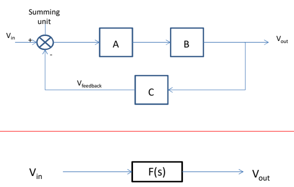
2. Feedback: $V_1 = V_{in} - V_{feedback}$

From the theory of feedback systems: $\frac{V_{out}}{V_{in}} = \frac{G(s)}{1+H(s)G(s)}$

3. Full transfer function:

$$F(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{G(s)}{1+G(s)F_C(s)} = \frac{4s(s-2)}{(s-j)(s+j)(s-2)+4s}$$

Now we can substitute the feedback system by one box



$$F_A(s) = 4$$

$$F_B(s) = \frac{s}{s^2+1}$$

$$F_C(s) = \frac{1}{s-2}$$

$$F(s) = \frac{4s(s-2)}{(s-j)(s+j)(s-2)+4s}$$

One can combine any number of units by such a manner

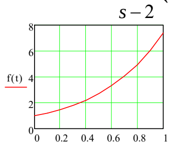
Stability – Устойчивость – Стійкість.

Results of transfer function $F(s)$ analysis

1. Functions $F(s)$ with ***real positive roots in denominator*** means continuously increasing output quantity. For example:

$$\frac{1}{s-2} \leftrightarrow e^{2t}$$

$$f(t) = e^{2t}$$



$@\ t = 0 \quad f(t) = 1$

$@\ t = \infty \quad f(t) = \infty$

This kind of $F(s)$ describes increasing energy, e.g. in generators. It is unacceptable in control systems.

Such function as ***F(s)=s+1*** [or a fraction ***A(s)/B(s)***], where the power of ***s*** in the nominator is the same or more than in the denominator, is typical for a system without energy losses (a loss-free system). It is impossible for a control system.

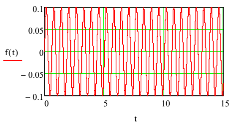
If you get it, stop and check again!

Results of transfer function $F(s)$ analysis

2. ***Imaginary roots*** in $F(s)$ means oscillation regime. For example, if:

$$F(s)=\frac{1}{s^2+100}\leftrightarrow f(t)=\frac{1}{10}\sin 10t$$

the system is in vibrational state (у коливальному стані) and does not tend to a fixed level.



This kind of system is unstable.

Normally control systems are designed by such a way to avoid oscillations

In some cases (very specific cases), the oscillations of small power can be introduced intentionally for improving the system operation.

Results of transfer function $F(s)$ analysis

3. ***Conjugate pair of imaginary roots*** (спряжена пара уявних коренів) in $F(s)$ means damped (decaying) oscillation (загасні коливання). For example:

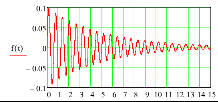
$$F(s)=\frac{1}{(s+0.2+j10)(s+0.2-j10)}=\frac{1}{s^2+0.4s+100.4}=\frac{1}{(s+0.2)^2+100}$$

Here the conjugate pair is: $s_1=-0.2-j10$ and $s_2=-0.2+j10$

This is a decaying sine process with time function:

$$f(t)=e^{-0.2t}\frac{\sin 10t}{10}$$

- an exponentially decaying sinusoid



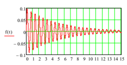
Such function can be the output quantity of a system that is activated by a step function. A system is applicable when the oscillations have already been damped.

Results of transfer function $F(s)$ analysis

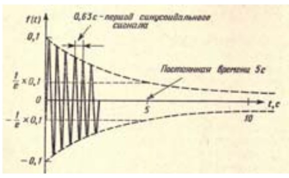
3. ***Conjugate pair of imaginary roots*** (continuation)

An exponentially decaying sinusoid

$$f(t)=e^{-0.2t}\frac{\sin 10t}{10}$$



Ringing «Звон»



Normally, it is desirable that this "ringing" was as short-time as possible. It would be better if it tends to 0.
For elimination of ringing and providing small response time, it is necessary to bring the system into critically damped mode.

System Stability

- From three features of typical response functions one can derive the criterion of system stability (for feedback systems – an open-loop system can't be unstable).
- Stability just means the absence of continuous oscillations.
- Transfer function (TF) of a Feedback System:

$$F_{FBS}(s)=\frac{G(s)}{1+H(s)G(s)}$$

with $G(s)$ is TF of the forward circuit (коло прямого сигналу)and $H(s)$ is TF of the feedback circuit (коло зворотного зв'язку).

System Stability

$$F_{FBS}(s)=\frac{G(s)}{1+H(s)G(s)}$$

- Thus, if the denominator $1+H(s)G(s)$ has roots, which contain ***positive or zero real components*** (додатні або нульові дійсні частини), then the time function that corresponds to $F_{FBS}(s)$ will have such members as $e^{\alpha t}\sin\omega t$ where α = positive value or 0, that is, members expressing continuous oscillation (sustained vibrations, незгасні коливання).
- The roots here are: $\alpha\pm j\omega t$. For example:

$$F_{FBS}(s)=\frac{1}{s^2+4s+13}=\frac{1}{(s-2-3j)(s-2+3j)}$$

then the roots are:

or:

$$s_1=2+3j,\quad s_2=2-3j$$

System Stability (cont.)

$$F_{FBS}(s)=\frac{G(s)}{1+H(s)G(s)}$$

$$s_{1,2}=2\pm 3j=\alpha\pm j\omega$$

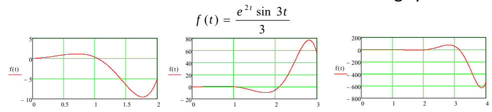
- The corresponding expression in time domain is

$$F_{FBS}(s)\leftrightarrow\frac{e^{2t}\sin 3t}{3}$$

- It consists of:
  - Continuously increasing exponential quantity
  - and sinusoid with ω=3, hence *f*=ω/2π=3/ 2π [Hz]

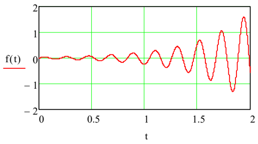
This function describes an unstable oscillating system:

$$f(t)=\frac{e^{2t}\sin 3t}{3}$$

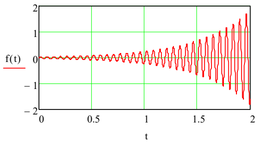


In case of higher frequency

$$F_{FBS}(t) = \frac{e^{2t}\sin 30t}{30}$$



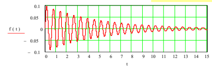
$$F_{FBS}(t) = \frac{e^{2t}\sin 100t}{30}$$



Criterion of stability

- The system is stable if the roots of the denominator of the transfer function $F_{out}(s)$ regarding to the entire system have negative real components.
- Система є стійкою (стабільною), якщо корені знаменника передавальної функції $F_{out}(s)$ всієї системи містять від'ємні дійсні частини.
- Система устойчива, если корни знаменателя передаточной функции $F_{out}(s)$ всей системы имеют отрицательные вещественные части.
- E.g. : $F_{out}(s) = \dfrac{1}{(s+0.2-j10)(s+0.2+j10)} \leftrightarrow \dfrac{e^{-\frac{1}{5}t}\sin 10t}{10}$

Roots: $s_1 = -0.2 - j10 \quad \text{and} \quad s_2 = -0.2 + j10$



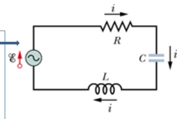
Oscillations are damped. And as t→∞, they fade completely.
The rate of damping depends on the real part of the root (-0.2, in this case)

- There were different criteria of stability developed
  - Routh–Hurwitz stability criterion – критерій Рута-Гурвица)
  - Bode Diagram (or Bode plot) – діаграма Боде
  - Root locus – кореневий годограф
- All of them are based on the same principal fact that the ***roots*** of the expression $1+H(s)G(s)$ ***with non-negative real parts*** are evidence of oscillations that characterize an unstable system.
- These roots are also named the ***poles*** of the transfer function.
- If at $s=s_1$ [where s_1 is a root of $1+H(s)G(s)$] the function $F_{out}(s) = \frac{G(s)}{1+H(s)G(s)}$ is approaching to infinity , s_1 is named the pole of the function $F_{out}(s)$.
- The roots of numerator $G(s)$ are called zeros s_{zero} of the function $F_{out}(s)$ because if $s = s_{zero}$, $F_{out}(s) \to 0$

Example

Given:

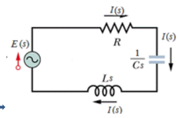
- Let we have a circuit
- Find :
  - Transfer function $\frac{i}{e} \leftrightarrow \frac{I(s)}{E(s)} = H(s)$
  - Dependence between R, L and C at critically damped mode.



Input and output values are function of time

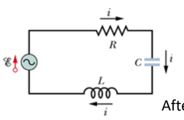
Solution:

- Two ways :
  - Applying differential equations, and their solution by Laplace method;
  - Using the equivalent Laplace diagram



Input and output values are function of *S*

1st way



Kirchhoff's voltage law:

$$e(t) = Ri(t) + \frac{1}{C}\int i(t)dt + L\frac{di(t)}{dt}$$

After derivation of both parts of the equation:

$$\frac{de(t)}{dt} = R\frac{di(t)}{dt} + \frac{1}{C}i(t) + L\frac{d^2 i(t)}{dt^2}$$

Applying Laplace method: $sE(s) = RsI(s) + \frac{1}{C}I(s) + Ls^2 I(s)$

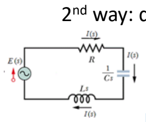
After factoring $I(s)$ out: $sE(s) = I(s)\cdot(Rs + \frac{1}{C} + Ls^2)$

Finally: $\dfrac{I(s)}{E(s)} = \dfrac{s}{Rs + 1/C + Ls^2} = H(s)$

The last eq. gives the Transfer Function for the given circuit.

However, the TF can be found directly from the equivalent diagram...

2nd way: directly from the equivalent diagram



According to Ohm's law (in Laplace domain):

$$I(s) = \frac{E(s)}{R + 1/Cs + sL} = \frac{E(s)}{\frac{Rs + 1/C + Ls^2}{s}}$$

$$\frac{I(s)}{E(s)} = \frac{s}{Rs + 1/C + Ls^2} = H(s) \qquad (*)$$

Now we can find $I(s)$ [and $i(t)$] for any driving source $E(s)$ [or $i(t)$]

We are interested in transient state, so let's $e(t)=U(t)$ – step function:

$U(t) \leftrightarrow \frac{1}{s}$. From (*): $I(s) = E(s)H(s) = E(s)\dfrac{s}{Rs + 1/C + Ls^2}$.

At $E(s) = \frac{1}{s}$ $\quad I(s) = \dfrac{1}{Rs + 1/C + Ls^2}$.

Now we can find the roots of square equation $Ls^2 + Rs + 1/C = 0$

$$Ls^2 + Rs + 1/C = 0$$

$$s^2 + \frac{Rs}{L} + \frac{1}{LC} = 0$$

$$s_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

If the expression under square root is negative, so the roots of equation are complex, and hence the current $i(t)$ contains a sine component

$$if \left(\frac{R}{L}\right)^2 - \frac{4}{LC} < 0 \rightarrow \frac{4}{LC} > \left(\frac{R}{L}\right)^2$$

$$s_{1,2} = a \pm jb$$

$$a = -\frac{R}{2L} \qquad b = \frac{1}{2}\sqrt{\frac{4}{LC} - \left(\frac{R}{L}\right)^2}$$

Note: the order of members is changed because j=√-1 is factorized. Then:

$$I(s) = \frac{1}{(s - a - jb)(s - a + jb)}$$

Imagine part of $s_{1,2}$ introduces a sine component $\sin bt$ into the current $i(t)$. If $b=0 \Rightarrow i(t)$ does not contain a sine component. Hence $b=0$ is a condition of critically damped mode:

$$b = 0 = \frac{1}{2}\sqrt{\frac{4}{LC} - \left(\frac{R}{L}\right)^2}$$

$$\frac{4}{LC} = \left(\frac{R}{L}\right)^2 \qquad R = 2\sqrt{\frac{L}{C}}$$

Now the conditions of each three possible responses (over-damped, sub-damped, and critically damped) are obvious.

Conditions of the modes for given example

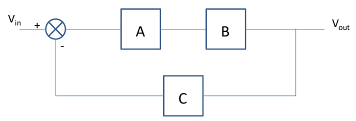
- Передемпфований over-damped $\quad \frac{4}{LC} > \left(\frac{R}{L}\right)^2$
- Недодемпфований sub-damped $\quad \frac{4}{LC} < \left(\frac{R}{L}\right)^2$
- Критично демпфований critically damped $\quad \frac{4}{LC} = \left(\frac{R}{L}\right)^2$

Note that here the real parts of the equation roots are always negative, that is, $a = -\frac{R}{2L}$.

Hence, the system is stable.

(the last is obvious because a feedback is absent)

Another example



$$A = A$$

$$B = \frac{1}{s+1}$$

$$C = \frac{2}{s+2}$$

Find:

1. Transfer function
2. Is this system stable at any values of A?
3. If A is a constant gain coefficient, what is its value A_{cr} to keep the system in the critically damped mode?
4. What will happen if A is two times more than A_{cr} ?